Applications of State Estimation in Aircraft Flight-Data Analysis

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This paper traces the evolution of the use of state estimation in the analysis of aircraft flight data and discusses some recent applications associated with airline turbulence encounters and high angle-of-attack flight tests. A unifying mathematical framework for state estimation is reviewed, and several examples are shown that illustrate a general approach for estimating variables that are difficult to measure. The diversity of the applications discussed and examples presented serve to demonstrate the potential advantages of using state estimation methods.

Introduction

ACCURATE determination of aircraft motions from noisy or incomplete measurements is an important problem in the analysis of flight-test experiments. The measurements often may contain significant errors which must be identified before the data are used in any performance calculations. Furthermore, direct measurements of certain important dynamic variables may be unreliable or impractical to perform. A similar problem occurs in the analysis of aircraft accidents, where the actual motions may have to be determined from a very limited data set. These problems are being solved by the analytical method known as state estimation.

Application of the state-estimation method to aircraft problems is possible because the forces and resulting motions of an aircraft along a flight path are related by well-known equations of motion. The equations may be used to produce estimates of force and motion variables (states) that can be compared with corresponding measurement time-histories, usually with iterations, until a suitable "match" is obtained.

The purpose of this paper is to provide information that will help the flight-data analyst recognize the potential advantages of using state estimation. The approach is to demonstrate by examples that seemingly diverse applications may be treated in a unified way by using one general-purpose state-estimation program for their solutions.

The paper proceeds as follows: first some historical background is presented which is followed by a discussion of three recently reported applications of state estimation for solving difficult measurement problems. The mathematical framework underlying the state-estimation method is then reviewed, and several examples are presented. Solutions to these example problems were obtained by using a state-estimation program developed at Ames Research Center. Each solution utilizes a different measurement set for the same simulated maneuver. In this way the robustness of the state-estimation procedure is demonstrated. Finally, some suggestions for future research are given in the concluding remarks.

Historical Background

The first application of state estimation to postflight data analysis can probably be attributed to the pioneering work of Otto Gerlach in the 1960s at the Delft Technological University, the Netherlands. This early contribution,[1,2] called "flight-path reconstruction" was primarily concerned with the accurate determination of angle of attack, pitch angle, and vehicle velocity during dynamic maneuvers. These "states" were obtained by integrating functions of measurements from the pitch-rate gyro and normal and longitudinal accelerometers. Initial conditions and bias terms were determined from airspeed and altitude measurements at the steady-state end points of the maneuver. The resulting "smoothed" time-histories were then used as a basis for subsequent parameter identification studies.

As Gerlach has pointed out, the technique of state estimation provides both a check on instrument accuracy and data consistency, and estimates of unmeasured or poorly measured variables. These items have been the primary objectives in most of the studies that followed Gerlach's original work. His students later improved and formalized the techniques that Gerlach had developed.[3,4] In this country, early advocates of the use of state estimation for flight-path reconstruction were Wingrove[5,6] at NASA, Eulrich and Weingarten[7] at Calspan, and Molusis[8] at Sikorsky Aircraft. Over the past few years, the work in this field has been evolving toward the use of more complete kinematic models, the development of more sophisticated algorithms, and the treatment of more difficult applications.

State estimation as a means of checking instrument accuracy and data consistency is now used by many flight-test groups.[9-21] Once a consistent, smoothed set of time-histories is obtained from the data, other analyses, such as identification of stability and control derivatives, are readily performed. In fact, relatively simple routines may be used for identification tasks, allowing the analyst freedom to develop a proper aerodynamic model. Since the data-consistency application has been extensively treated in the literature, it will not be discussed further here. Instead, the paper will address some of the more recent applications of aircraft state estimation in obtaining estimates of difficult-to-measure variables.

Recent Applications

Three recently reported applications of state estimation for determining unmeasured or poorly measured flight variables are reviewed in this section. The applications, quite diverse in terms of the available measurements and desired estimates, indicate the wide range of problems that can be solved with state-estimation techniques. The variety in estimation techniques used to treat the applications described here should not lead the analyst to feel that each such problem requires a special ad hoc approach. As will be shown later, these and other estimation applications may be treated in a unified way; that is, they may all be solved using one general-purpose state-estimation program.

Presented as Paper 83-2087 at AIAA Atmospheric Flight Mechanics Conference, Gatlinburg, Tenn., Aug. 15-17, 1983; received Nov. 14, 1983; revision received Jan. 24, 1985.

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For determining flight trajectories in aircraft accident analyses, state estimation can be effectively used to combine data from several sources (radar site, flight recorder, etc.) to determine motions along a trajectory.[22] In addition, the winds along a flight trajectory can often be estimated. This has been useful in the analysis of recent airline clear-air turbulence upsets, which are a continuing problem. A paper by Parks et al.[23] describes the estimation of winds by using data from a DC-10 encounter with severe high-altitude turbulence. The wind estimates from that analysis led Parks to hypothesize the presence of a classical "cat's-eye" vortex phenomenon in the jetstream shear layer at the time of the encounter.

Data from a digital flight recorder like the one carried by a DC-10 includes accelerations, Euler angles, altitude, and airspeed, sampled at intervals of 0.25-1.0 s. Enough additional information is available to approximate the records of angles of attack and sideslip. The addition of ground-based air traffic control (ATC) radar records provides a measurement set approaching that available from flight test. To obtain the desired wind estimates, Parks first transformed the accelerations into an Earth frame, then integrated to obtain aircraft velocity with respect to the Earth. A consistent set of initial conditions and accelerometer bias corrections was obtained by matching calculated position time-histories with radar and barometric altitude records. The wind components were then found as the difference between the aircraft velocities with respect to Earth and air mass in the Earth frame.

Other applications of state estimation that are becoming increasingly important are associated with the testing of high-performance aircraft. In large angle-of-attack maneuvers and spin tests, for example, measurements of Euler angles, airspeed, and aerodynamic angles (angles of attack, sideslip) may contain significant errors. In a recent paper, Taylor[24] discussed the estimation of Euler-angle time-histories and air-data bias and scale-factor errors for a spinning airplane. The measurement set for this application consisted of accelerometer, rate-gyro, and air-data measurements, and knowledge of the winds. With the measured accelerations and angular velocities as forcing functions, Taylor "fitted" the air-variable measurement records, using a squared-error criterion and a Newton-Raphson algorithm to determine the desired estimates of bias and scale-factor errors and Euler-angle time-histories. To avoid possible singularities in angle calculations, Taylor utilized the differential equations relating the angular velocities and direction cosines.

For some large angle-of-attack maneuvers, merely estimating bias and scale-factor errors for the air data may not be sufficient. In a paper describing the identification of indicial functions, Gupta and Iliff[25] found it necessary to obtain estimates for air-variable time-histories for the high angle-of-attack flight-test regime. The data used in the solution of this problem consisted of on-board measurements of Euler angles as well as radar tracking data (slant range and bearing and elevation angles). Winds were estimated during low angle-of-attack portions of the test when air data were usable. The winds were then assumed to remain constant during those test segments when the air variables were to be estimated. The estimates were obtained by "smoothing" the radar data for the Earth-frame components of aircraft velocity, subtracting the winds, and then transforming to the aircraft body-frame system and calculating the desired estimates of airspeed, angle of attack, and angle of sideslip.

The applications discussed in this section provided motivation for the development of several examples to be presented later, including one showing that Euler angles and air variables may all be estimated using inertial position measurements, accelerometer and rate-gyro data, and knowledge of the winds. The examples illustrate a general approach that may be used to treat many problems in the analysis of aircraft flight data. First, however, certain essential aspects of the mathematics of model and algorithm development will be reviewed.

Mathematical Framework

In the context of this paper, the term state estimation refers to a process that solves a state model

$$\dot{x}=f(x,w), \qquad x(t_0)=x_0 \tag{1}$$

such that a measurement model

$$z=h(x)+v \tag{2}$$

suitably matches the available data records over an interval (t_0,t_f), usually in a least-squared error or minimum-variance sense. In Eq. (1), x is an n_x-element state vector, and w is an n_w-element forcing-function vector; in Eq. (2) z and v are n_z-element vectors representing the measurements and corresponding (random) measurement errors. For aircraft problems, the state and measurement models together represent the dynamics of a rigid body in order to generate time-histories of the variables needed in the estimation process. These variables may include vehicle Euler angles, angular rates, and linear accelerations, as well as slant range and bearing (radar data), pressure altitude and airspeed, and angles of attack and sideslip (air data). Any bias or scale-factor errors associated with the state or measurement models are appended to the state vector and treated as constant but unknown parameters.

The problem of postflight state estimation considered in this paper is usually defined as a fixed-interval smoothing problem. For the case in which all the elements of the forcing-function vector w in Eq. (1) are found in the measurement set, the problem consists of determining the x_0 that minimizes the squared-error performance measure

$$J=\frac{1}{2}(x_0-\bar{x}_0)^t P_0^{-1}(x_0-\bar{x}_0) + \frac{1}{2}\int_{t_0}^{t_f}[z-h(x)]^t R^{-1}[z-h(x)]\,\mathrm{d}t \tag{3}$$

subject to the dynamic constraint of Eq. (1). In Eq. (3), $\bar{x}_0$ is an a priori estimate of x_0, and P_0 and R are weighting matrices. Note that the first term of Eq. (3) serves as a "penalty" function and tends to bias the estimate of x_0 toward the a priori value. A Newton-Raphson approach for minimizing Eq. (3) performs very well. One algorithm used by some workers in the field[5,18,24] involves choosing an x_0, solving Eqs. (1) and (2) and the additional "sensitivity" equations

$$S=h_x\Phi(t,t_0); \quad \dot{\Phi}=f_x\Phi; \quad \Phi(t_0,t_0)=I \tag{4}$$

and then evaluating a change in initial condition given by

$$\delta x_0=-\left[P_0^{-1}+\int_{t_0}^{t_f}S^t R^{-1} S\,\mathrm{d}t\right]^{-1} \times\left[P_0^{-1}(x_0-\bar{x}_0)+\int_{t_0}^{t_f}S^t R^{-1}[z-h(x)]\,\mathrm{d}t\right] \tag{5}$$

iterating until J reaches a minimum. The matrices in Eq. (4) are defined as

$$h_x=\frac{\partial h}{\partial x}; \quad f_x=\frac{\partial f}{\partial x}; \quad \Phi=\frac{\partial x}{\partial x_0} \tag{6}$$

Note that the computational burden for each iteration of the algorithm involves the solution of $n_x\times(n_x+1)$ differential equations and $n_x\times(n_x+3)/2$ quadratures. A mathematically equivalent but a more efficient algorithm for computing δx_0 is given by[14]

$$\delta x_0=-(P_0^{-1}+M_0)^{-1}[P_0^{-1}(x_0-\bar{x}_0)+\alpha_0] \tag{7}$$

where

$$\dot{M} = -f_x^t M - M f_x - h_x^t R^{-1} h_x, \qquad M(t_f) = 0$$

$$\dot{\alpha} = -f_x^t \alpha + h_x^t R^{-1} [z - h(x)], \qquad \alpha(t_f) = 0 \tag{8}$$

In Eq. (8), M is the $n_x \times n_x$ "information" matrix and α is the "gradient" vector, and in Eq. (7), $M_0 = M(t_0)$, $\alpha_0 = \alpha(t_0)$. This algorithm requires the solution of only $n_x \times (n_x + 5)/2$ differential equations at each iteration.

For the important case in which some elements of the forcing function are poorly measured or are missing altogether from the measurement set, the state-estimation problem may be recast to include an unknown forcing function in the estimation process. This can be done simply by augmenting the performance measure of Eq. (3) with an additional term so that

$$J' = J + \frac{1}{2} \int_{t_0}^{t_f} w^t Q^{-1} w \, \mathrm{d}t \tag{9}$$

where Q is a weighting matrix. Now the fixed-interval smoothing problem consists of determining x_0 and w to minimize Eq. (9) subject to the dynamic constraint [Eq. (1)]. The necessary conditions for the minimization of Eq. (9) lead to a nonlinear, two-point boundary value problem.[26] A "sensitivity"equation approach to its solution, analogous to the algorithm of Eqs. (4) and (5), is unstable. An extension of the algorithm of Eqs. (7) and (8), however, is stable and gives good results.[14,17] This method involves choosing x_0 and w, solving Eqs. (1) and (2) to obtain a "nominal" solution, and then determining a change in the initial condition by evaluating Eq. (7), where M_0 and α_0 are found by solving the backward "information" filter:

$$\dot{M} = -f_x^t M - M f_x - h_x^t R^{-1} h_x + M f_w Q f_w^t M, \quad M(t_f) = 0$$

$$\dot{\alpha} = -f_x^t \alpha + h_x^t R^{-1} [z - h(x)] + M f_w (Q f_w^t \alpha + w), \quad \alpha(t_f) = 0 \tag{10}$$

The change in the unknown forcing function can then be determined by solving a first-order expansion of the dynamic constraint about the nominal solution

$$\delta\dot{x} = f_x \delta x + f_w \delta w, \qquad \delta x(t_0) = \delta x_0 \tag{11}$$

in the forward direction, with

$$\delta w = -w - Q f_w^t (\alpha + M \delta x) \tag{12}$$

This procedure is iterated until the performance measure of Eq. (9) is minimized. Note that this algorithm reduces to the Newton-Raphson algorithm of Eqs. (7) and (8) if there are no unknown forcing functions.

An alternative and much-used method for state estimation with unknown forcing functions employs an extended Kalman filter coupled with a backward smoother.[9-13,15,16,20,21] Although this procedure gives an approximate solution to the two-point boundary value problem, the results of some test cases have indicated close agreement with results obtained using the "exact" algorithm of Eqs. (7) and (10-12). The latter algorithm is employed in an aircraft state-estimation program developed at Ames Research Center called SMACK (smoothing for aircraft kinematics). It has the following advantages over the extended Kalman filter method:

1) Convergence of the algorithm is easily monitored during minimization of the performance measure of Eq. (9), permitting an assessment of the "robustness" of any solution.

2) Complete uncertainty about the a priori estimate is established by setting the weighting matrix $P_0^{-1} = 0$ in Eq. (7), thus avoiding any biasing of solution parameters.

3) A decoupling of constant and variable elements of the state vector is obtained by using the information filter of Eq. (10), resulting in a simple calculation procedure.

Application Examples

In this section, the four examples based on the applications of state estimation reviewed in a preceding section and one example of an application not previously reported are developed and discussed. The solution of each example was obtained by the same general-purpose state-estimation program. In this way, the full potential of the method for solving difficult measurement problems can be appreciated. Data for each example were taken from a simulated trajectory consisting of a rising, coordinated, 180-deg turn in the presence of wind. Small amounts of random noise, usually 1% or less, were added to each measured variable, and all measurements were recorded once per second. A summary of the available measurements and variables to be estimated for each example is given in Table 1. The SMACK program was used to provide the estimates. A brief description of the state and measurement models [Eqs. (1) and (2)] used in the program follows now.

Aircraft motions are assumed to be governed by a six-degree-of-freedom kinematic model, referred to a flat, nonrotating Earth.[27] The usual choice of state variables leads to a formulation in which both state and measurement models are nonlinear. However, by choosing a set of state variables consisting of Euler angles (ϕ,θ,ψ), inertial position (x,y,h) and their time derivatives $(\dot{\phi},\dot{\theta},\dot{\psi})$. $(\ddot{\phi},\ddot{\theta},\ddot{\psi})$; $(\dot{x},\dot{y},\dot{h})$, $(\ddot{x},\ddot{y},\ddot{h})$, a sim-

Table 1 Measured and estimated variables used in example applications of state estimation

Variables in state-estimation procedure	Example 1 (Ref. 23)	2 (Ref. 24)	3 (Ref. 25)	4	5
Linear accelerations (a_x,a_y,a_z)	Measured	Measured		Measured	Measured
Angular velocities (p,q,r)		Measured			Measured
Vehicle positions (R,B,h)	Measured		Measured	Measured	Measured
Winds $(W_{xy},W_{\mathrm{hd}},w_h)$	Estimated	Measured	Measured	Measured	Measured
Euler angles (ϕ,θ,ψ)	Measured	Estimated	Measured	Measured	Estimated
Air variables (V,α,β)	Measured	Measured	Estimated	Estimated	Estimated

ple linear state model is realized for SMACK:

$$\begin{bmatrix} \dddot{\phi} \\ \dddot{\theta} \\ \dddot{\psi} \end{bmatrix} = \begin{bmatrix} d_\ell \\ d_m \\ d_n \end{bmatrix}; \qquad \begin{bmatrix} \dddot{x} \\ \dddot{y} \\ \dddot{h} \end{bmatrix} = \begin{bmatrix} d_x \\ d_y \\ d_h \end{bmatrix} \tag{13}$$

Here (d_ℓ, d_m, d_n) and (d_x, d_y, d_h) make up a vector of forcing functions to be estimated. To include motion of the air mass, the state model is augmented with wind states (w_x, w_y, w_h) so that

$$\begin{bmatrix} \dot{w}_x \\ \dot{w}_y \\ \dot{w}_h \end{bmatrix} = \begin{bmatrix} g_x \\ g_y \\ g_h \end{bmatrix} \tag{14}$$

and (g_x, g_y, g_h) are added to the vector of forcing functions. As will be demonstrated, all nonlinearities associated with aircraft kinematics appear in the measurement model. With this formulation, the partial-derivative matrix for the state model [f_x in Eq. (6)] is constant along any trajectory, a feature that significantly improves the efficiency of the SMACK algorithm.[28]

Except for attitudes (ϕ, θ, ψ) and positions (x, y, h), all elements of the measurement model are nonlinear functions of the state variables. The angular velocities, for example, are expressed in terms of the states as

$$\begin{aligned} p &= \dot{\phi} - \dot{\psi}\sin\theta \\ q &= \dot{\theta}\cos\phi + \dot{\psi}\sin\phi\cos\theta \\ r &= -\dot{\theta}\sin\phi + \dot{\psi}\cos\phi\cos\theta \end{aligned} \tag{15}$$

To obtain the aircraft body-axis velocities (u_a, v_a, w_a) and specific forces (a_x, a_y, a_z) requires the coordinate transformations

$$\begin{bmatrix} u_a \\ v_a \\ w_a \end{bmatrix} = L \begin{bmatrix} \dot{x} - w_x \\ \dot{y} - w_y \\ \dot{h} - w_h \end{bmatrix}; \qquad \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = L \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{h} + g \end{bmatrix} \tag{16}$$

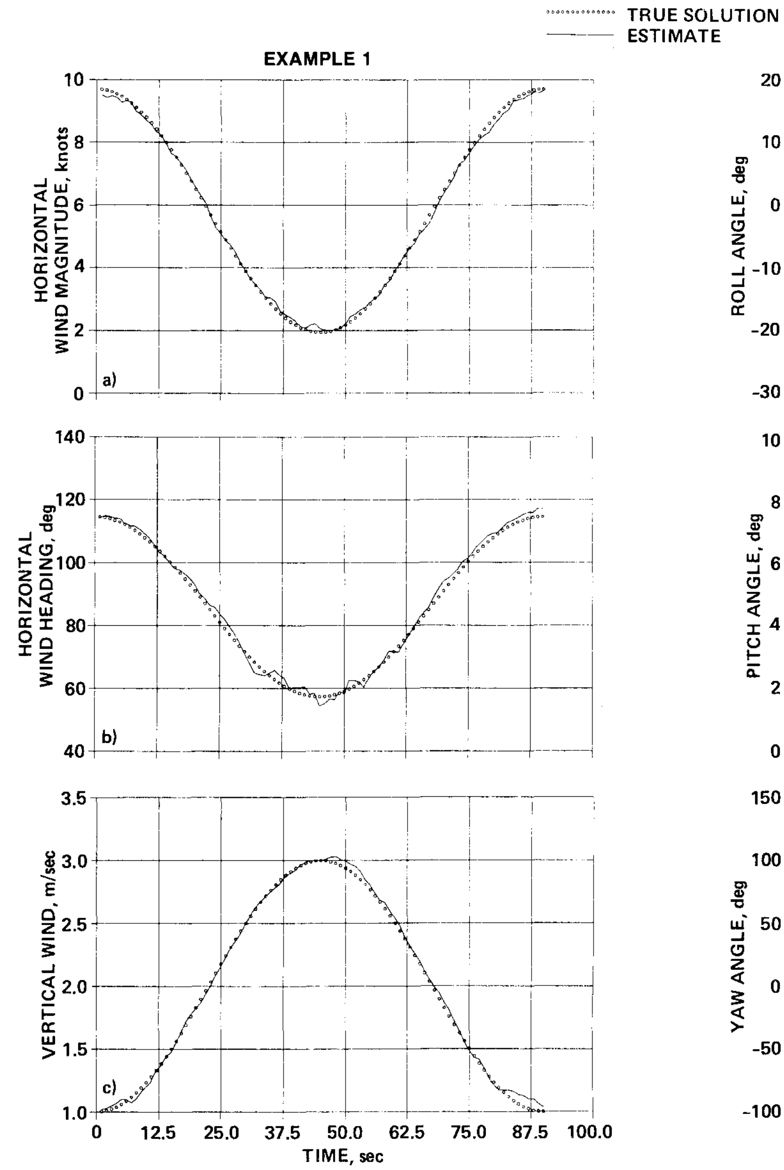

Fig. 1 Estimated and true winds: a) horizontal wind magnitude, b) horizontal wind heading, and c) vertical wind.

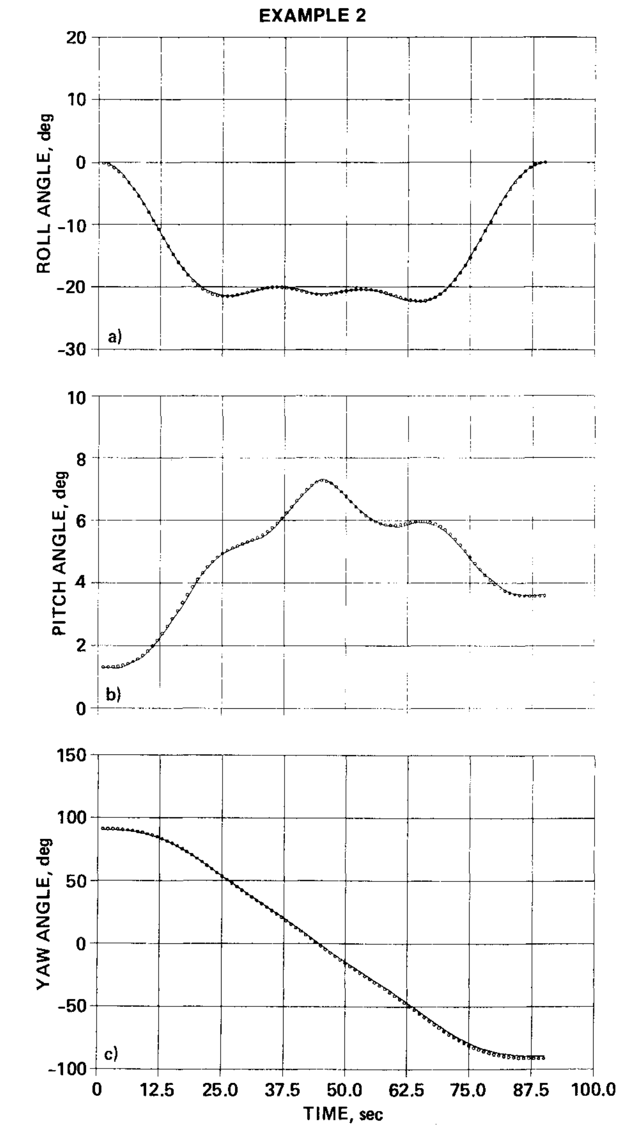

Fig. 2 Estimated and true Euler angles: a) roll angle, b) pitch angle, and c) yaw angle.

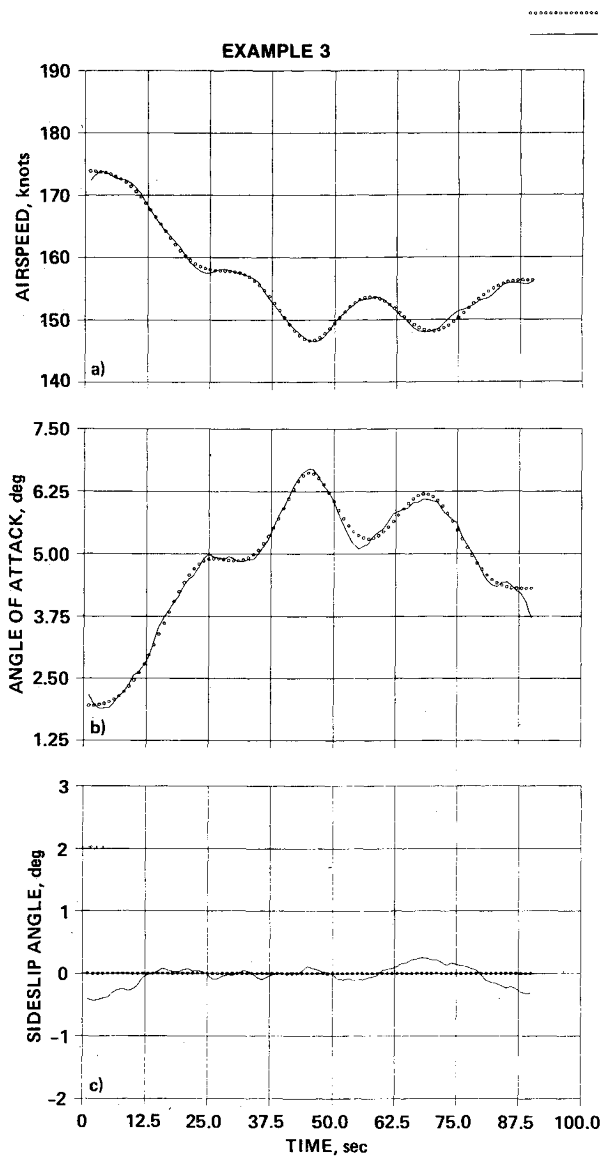

Fig. 3 Estimated and true air variables: a) true airspeed, b) angle of attack, and c) sideslip angle.

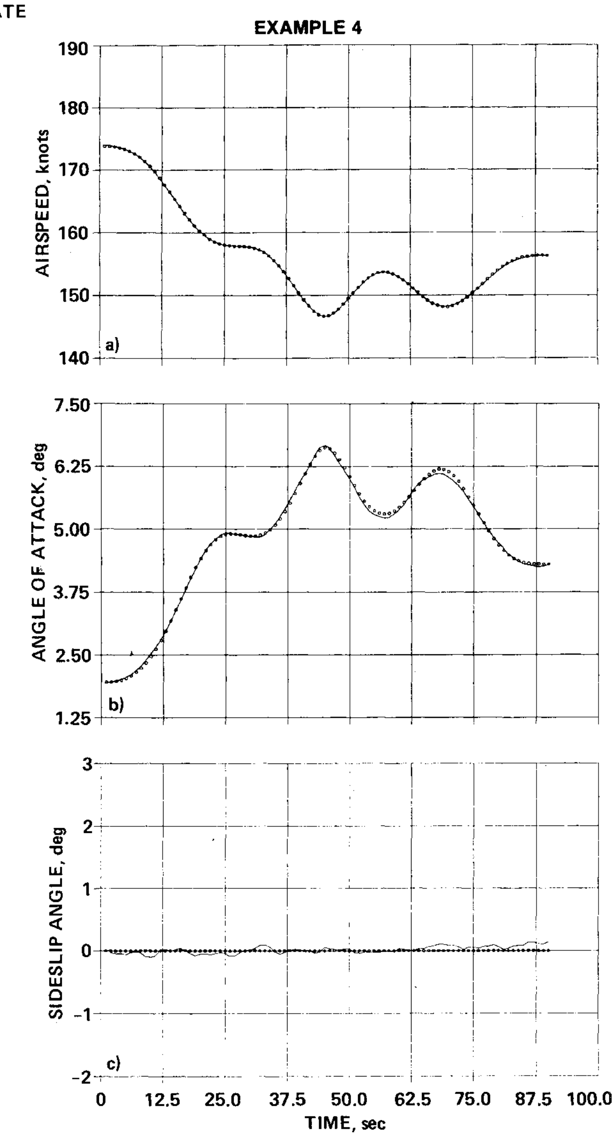

Fig. 4 Estimated and true air variables: a) true airspeed, b) angle of attack, and c) sideslip angle.

respectively, where the transformation is defined by the direction-cosine matrix

$$L = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & \sin\theta \\ \sin\phi\sin\theta\cos\psi & \sin\phi\sin\theta\sin\psi & \\ -\cos\phi\sin\psi & +\cos\phi\cos\psi & -\sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi & \cos\phi\sin\theta\sin\psi & \\ +\sin\phi\sin\psi & -\sin\phi\cos\psi & -\cos\phi\cos\theta \end{bmatrix} \tag{17}$$

The measurement set may also include aerodynamic variables (airspeed V, angles of attack α and sideslip β)

$$V = (u_a^2 + v_a^2 + w_a^2)^{1/2}; \quad \alpha = \tan^{-1}(w_a/u_a); \quad \beta = \sin^{-1}(v_a/V) \tag{18}$$

and ground-based radar information such as slant range R and bearing angle B, where

$$R = (x^2 + y^2 + h^2)^{1/2}; \qquad B = \tan^{-1}(y/x) \tag{19}$$

Finally, the horizontal wind estimates (magnitude W_{xy} and heading $W_{\rm hd}$) are computed from the relations

$$W_{xy} = (w_x^2 + w_y^2)^{1/2}; \qquad W_{\rm hd} = \tan^{-1}(-w_y/-w_x) \tag{20}$$

The first example illustrates a wind-estimation application and uses the measurement set available to Parks et al.[23] in their analysis of the DC-10 turbulence upset (see Table 1). In the analysis of this problem by SMACK, all elements of the forcing-function vector were estimated, and the measurement time-histories (a_x, a_y, a_z), (ϕ, θ, ψ), (V, α, β), and (R, B, h) were fitted in the least-squared-error procedure. The resulting wind estimates are shown in Fig. 1, along with the "true" winds for comparison. The close agreement of the horizontal wind records indicated in Table 2 is probably better than could be expected in practice, since ATC en route radar data are recorded only about once every 10 s.

A second example illustrates the application of state estimation for determining Euler angles using the measurement set of Taylor[24] as summarized in Table 1. In the analysis by SMACK, the inertial wind components (w_x, w_y, w_h) needed in Eq. (16) were obtained from measured winds. The measurements (a_x, a_y, a_z), (p, q, r), and (V, α, β) were fitted, with bias and scale-factor estimates obtained for the air-data records. The Euler-angle estimates are shown in Fig. 2, plotted with the corresponding true values. Estimation errors are given in Table 2. It should be noted that the pitch-angle excursion is not large along the simulated trajectory; therefore, Eq. (13) is adequate for determining the Euler angles. For extreme maneuvers, in which the pitch angle may approach 90 deg, it is necessary to avoid ambiguity in attitude determination. The use of quaternions for this purpose is currently under investigation.

The measurement set employed by Gupta and Iliff[25] for the estimation of air-data variables, shown in Table 1, is the basis for the third example. In the solution of this example, the wind components are again assumed to be known. Here the measurements fitted were (ϕ,θ,ψ) and (R,B,h). The results of the solution for the air variables (V,α,β) are shown in Fig. 3 and Table 2. In an application such as this, the radar data-sample rate may not be high enough to provide sufficient air-variable estimates (in the Gupta case the sample rate was 1/s). It may be both necessary and convenient to augment the measurement set with on-board accelerometer data. Example 4 illustrates this case by including (a_x,a_y,a_z) in the measurement set to be fitted. The results are shown in Fig. 4 and Table 2, where a comparison can be made with the results of the preceding example.

It is interesting to note that the Taylor application requires air-data measurements, whereas the Gupta-Iliff application requires Euler-angle measurements. It would seem useful for some extreme flight-test situations to be able to estimate both sets of variables. That this can be accomplished by state estimation is illustrated by a final example (example 5). As indicated in Table 1, this procedure utilizes radar position data (including altitude), knowledge of the winds, and measurements of the "strap-down" variables (linear accelerations and angular velocities). Results of a simulation experiment as obtained by SMACK are shown in Fig. 5, where good correspondence between estimated and true time-histories can be observed. A comparison of the estimation accuracy obtained here with the results of the three previous experiments can be seen in Table 2.

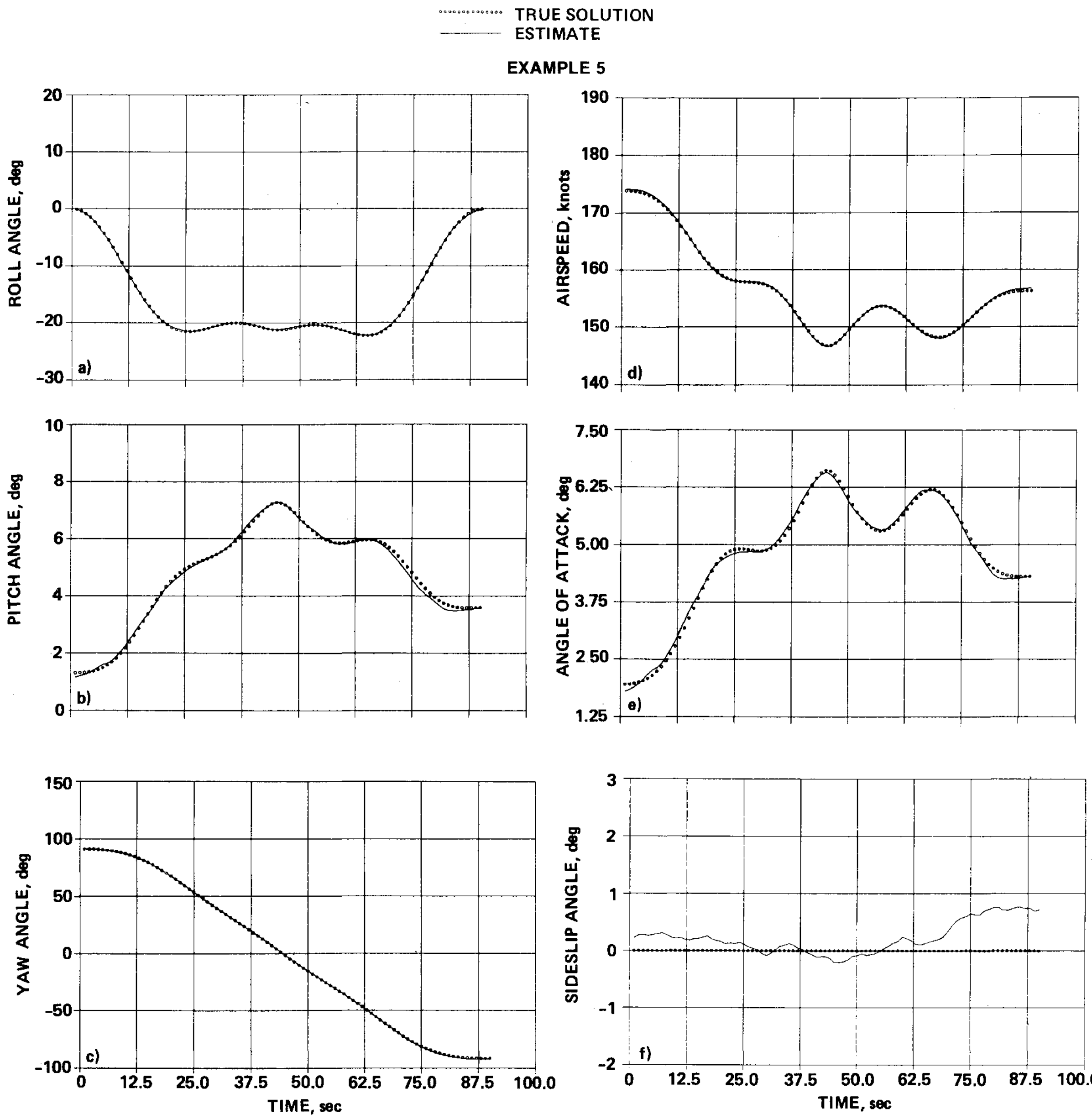

Fig. 5 Estimated and true Euler angles and air variables: a) roll angle, b) pitch angle, c) yaw angle, d) true airspeed, e) angle of attack, and f) sideslip angle.

Table 2 Estimation results for example applications of state estimation[a]

Estimated variables	Example 1 (Ref. 23) Mean (dev)	2 (Ref. 24) Mean (dev)	3 (Ref. 25) Mean (dev)	4 Mean (dev)	5 Mean (dev)
Winds					
Horizontal magnitude, knots	0.026 (0.115)				
Horizontal heading, deg	−0.836 (1.450)				
Vertical, m/s	−0.014 (0.032)				
Euler angles					
Roll, deg		−0.050 (0.103)			0.004 (0.071)
Pitch, deg		0.025 (0.035)			−0.035 (0.088)
Yaw, deg		−0.746 (0.881)			0.238 (0.285)
Air variables					
Airspeed, knots			0.082 (0.442)	0.021 (0.054)	−0.047 (0.156)
Angle of attack, deg			0.021 (0.109)	0.013 (0.045)	−0.001 (0.070)
Sideslip, deg			0.036 (0.161)	−0.015 (0.056)	−0.224 (0.276)

[a]Mean and dev (deviation) refer to error between estimated and true time-histories.

Concluding Remarks

In this paper the evolution of state-estimation technology for use in analyses of aircraft flight data has been traced. Three recent applications involving estimation of winds, Euler angles, and air variables have been discussed; some pertinent mathematical features of the state-estimation procedure have been reviewed; and several examples have been presented. The examples demonstrate that a general-purpose state-estimation program can be used to solve the applications discussed. In one additional example of an application not previously reported, it has been shown that inertial position measurements and on-board strap-down measurements can be combined using a state-estimation procedure to provide estimates of Euler angles and air-data variables. It is hoped that the paper has helped to make the flight-data analyst aware of the potential advantages of using state estimation in solving difficult measurement problems.

Several interesting directions for further research exist. Among these is the possible implementation of information-retrieval systems that are specifically designed to be used with state-estimation methods. As the application examples described in this paper have shown, difficult-to-measure variables can instead be estimated, using other variables that can be reliably measured over wide ranges of vehicle operation. Although the calculations for these estimation problems were performed postflight (off-line), it would be reasonable to assume that for some situations on-line filtering versions might be both desirable and feasible. Finally, there still is work to be done to make the state-estimation algorithms more efficient and less susceptible to vehicle attitude singularities during extreme maneuvers. Current efforts include the use of quaternions in the nonlinear dynamic model, and coordinate transformations that provide an equivalent linear dynamic model.

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